Large Scale Hypothesis Testing

CSE545 - Spring 2023

Stony Brook University

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Big Data Analytics, The Class

Goal: Generalizations
A *model* or *summarization* of the data.

Data Workflow Systems

Hadoop File System ✓
Spark ✓
Streaming ✓
MapReduce ✓
Tensorflow ✓

Algorithms and Analyses

Similarity Search ✓
Large Scale Hyp. Testing
Link Analysis ✓
Recommendation Systems
Deep Learning

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The Data Whisperer

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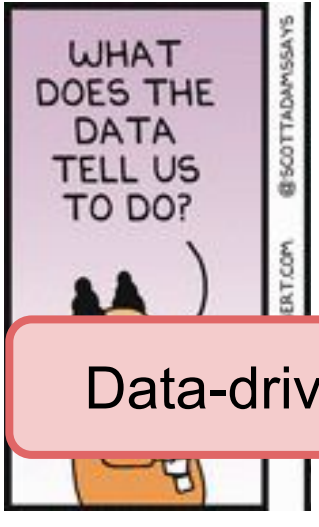

Data-driven (evidence-based) **decision**

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Discrete **Finding(s)**
$\mathcal{F}$ is (likely) True

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Blue cell phones cases are selling the most.

The ResImageGenNet model is most accurate.

Those >70 have a greater mortality rate from the viral infection.

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Hypotheses!
Potential findings -- to be tested for happenstance.

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Hypothesis Testing

Also known as… “Don’t be Dilbert’s Boss!”

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Formally, two types:

H_0*: null hypothesis* -- some “default” value; “null”: nothing changes

H_1: the alternative -- the opposite of the null => a change or difference

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Goal: Make sure what we observed was unlikely to happen by chance.

Thus, we want to know:

Given null, what is the probability of the observation or worse

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-> If low enough, then we “reject the null (H_0) in favor of H_1.”

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H_0: The blue case is not selling more than average.

The Hypothesis Test "Algorithm"

observations (i.e. data)

level of significance

Input: H_0, obs, α

Output: decision

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The Hypothesis Test "Algorithm"

observations (i.e. data)

level of significance

Input: H_0, obs, α

probability of what we observed or worse (i.e. more extreme)

$p(x>=obs \mid H_0) < \alpha$

Output: decision

H_0: The blue case is not selling more than average.

The Hypothesis Test "Algorithm"

```
Input: H0, obs, α




if p(x>=obs | H0) < α:
    decision = "Reject H0!"
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Conditional is sometimes evaluated indirectly by first finding the "critical value" of some ***measurement*** *such that:*
if measurement > critical_value then p(obs|H0) < α

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Probability Distributions: Review

$\mathbf{X}$: A mapping from $\mathbf{\Omega}$ to $\mathbb{R}$ that describes the question we care about in practice.

$\mathbf{X}$ is a *continuous random variable* if it can take on an infinite number of values between any two given values.

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Error of RedImageGenNet Classifier

Amount of sales of a blue case

Continuous Distributions

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X is a *continuous random variable* if there exists a function *fx* such that:

$$f_X(x) \geq 0, \text{for all } x \in X,$$

$$\int_{-\infty}^{\infty} f_X(x)dx = 1, \quad \text{and}$$

$$\mathrm{P}(a < X < b) = \int_a^b f_X(x)dx$$

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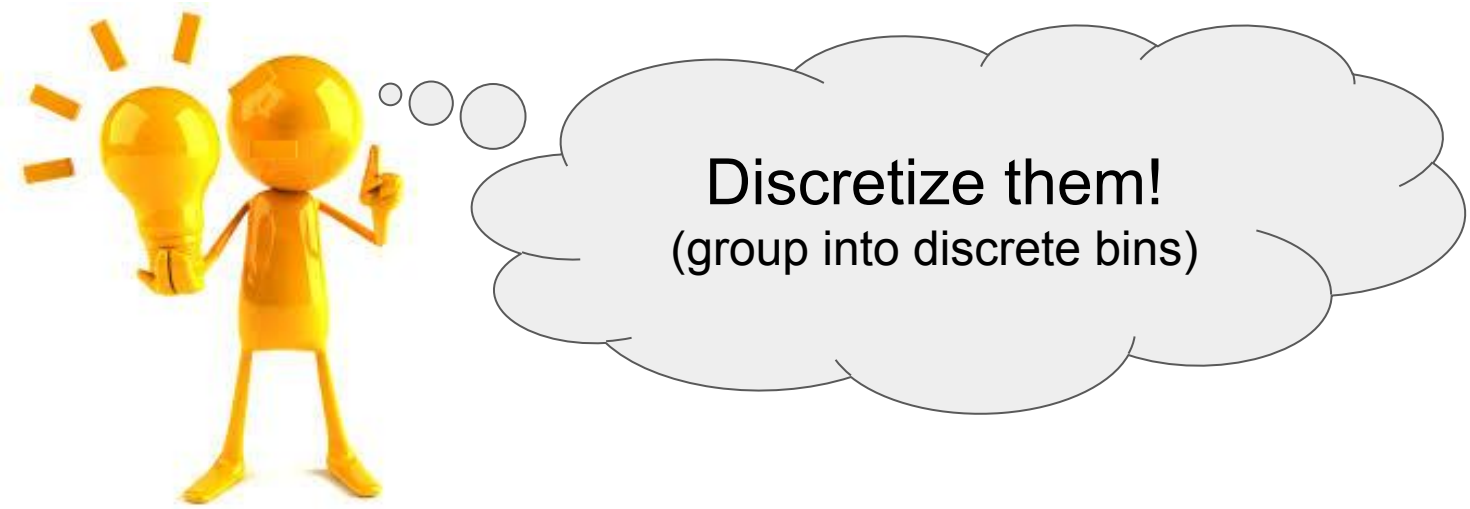
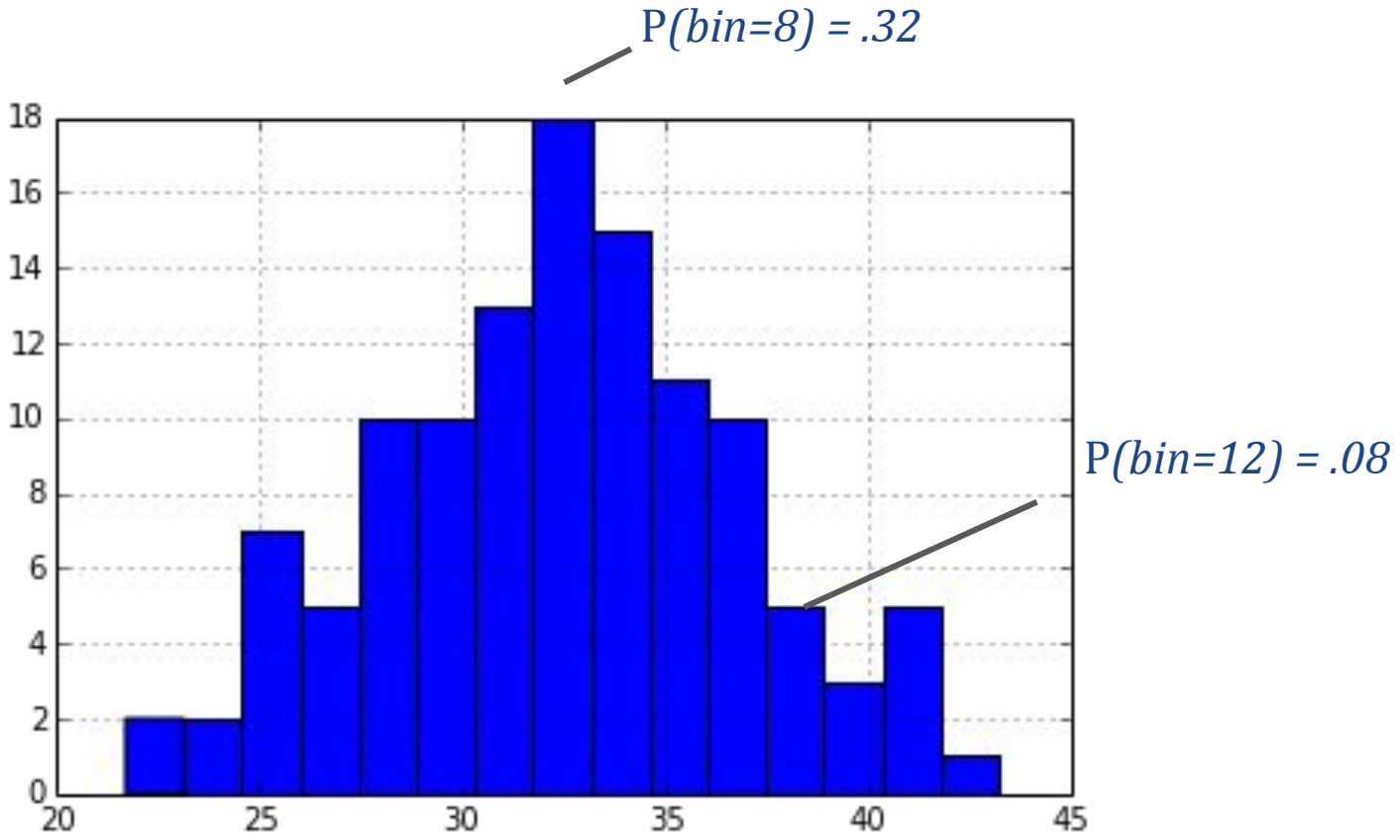
***fx* : "probability density function" (pdf)**

Continuous Distributions



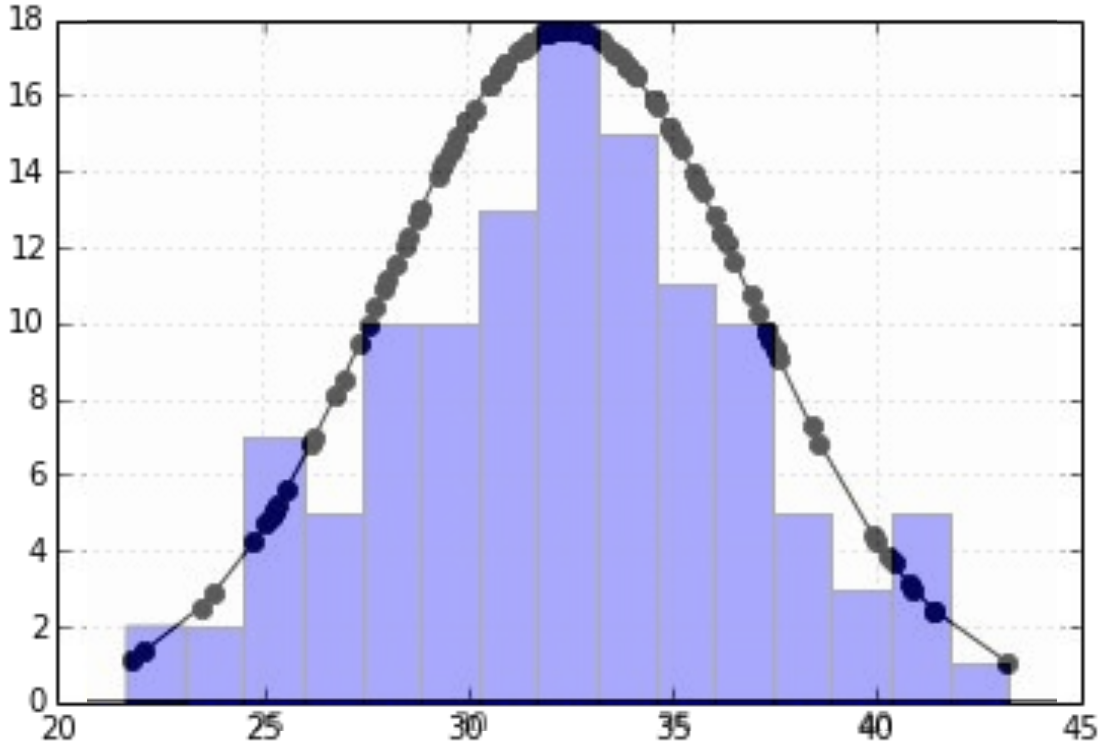
How to model?

Continuous Distributions



But aren't we throwing away information?

Continuous Distributions



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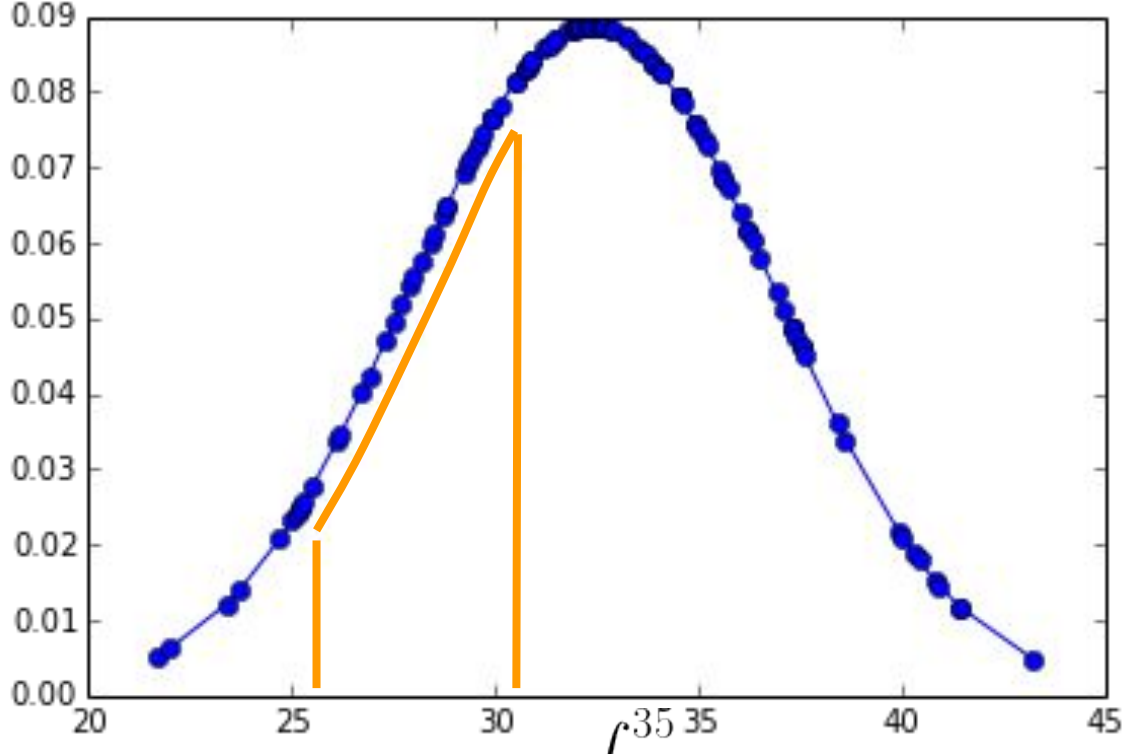
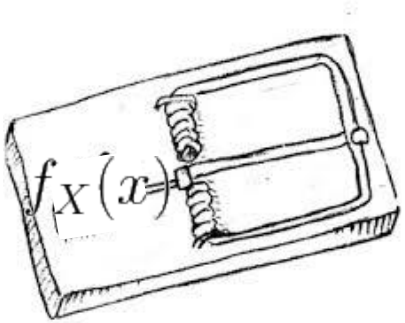


$$\mathrm{P}(25 < X < 35) = \int_{25}^{35} fx(x)dx$$

Continuous Distributions

Common Trap

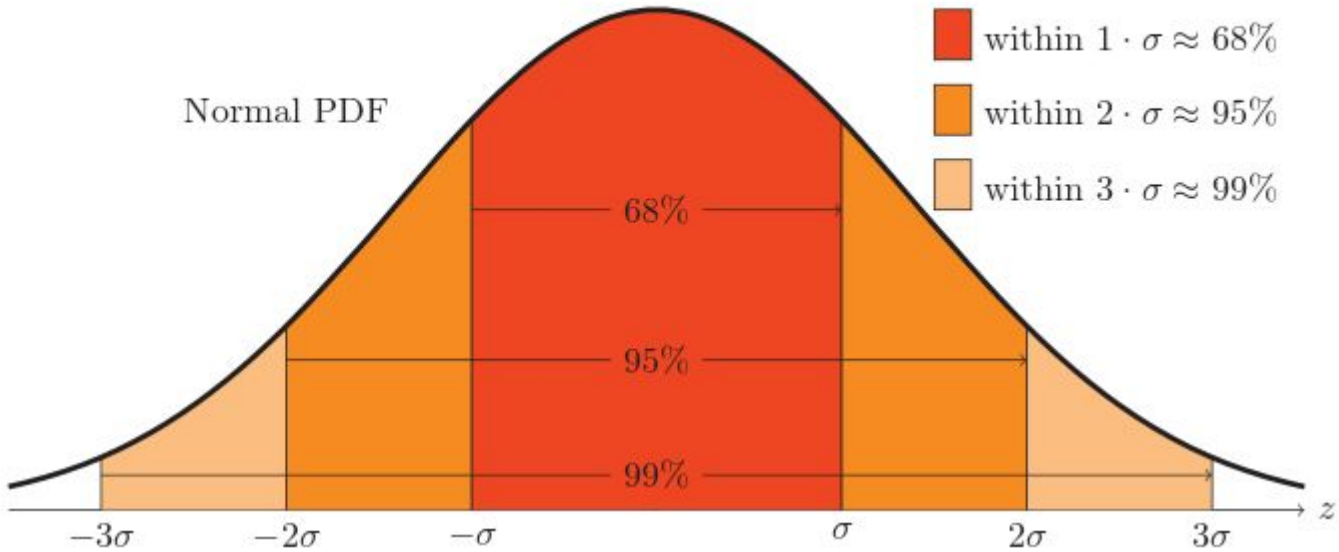
- $f_X(x)$ does not yield a probability
  - $\int_a^b f_X(x)dx$ does
  - x may be anything ($\mathbb{R}$)
    - thus, $f_X(x)$ may be > 1



Continuous Distributions

Common *pdf*s: $\text{Normal}(0, 1)$

$$P(-1 \le Z \le 1) \approx .68, \quad P(-2 \le Z \le 2) \approx .95, \quad P(-3 \le Z \le 3) \approx .99$$



Credit: MIT Open Courseware: Probability and Statistics

Continuous Distributions

Common *pdf*s: Normal(0, 1) ("standard normal")

How to "standardize" any normal distribution:

1. subtract the mean, μ (aka "mean centering")
2. divide by the standard deviation, σ

$z = (x - \mu) / \sigma$, (aka "z score")

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Amount of sales of a blue case

Discrete Random Variables

For a given *discrete* random variable X, *probability mass function* (*pmf*), *fx:* $\mathbb{R} \rightarrow [0, 1]$, is defined by:

$$f_X(x) = \mathrm{P}(X = x)$$

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Was a single sale a blue case: {0, 1}

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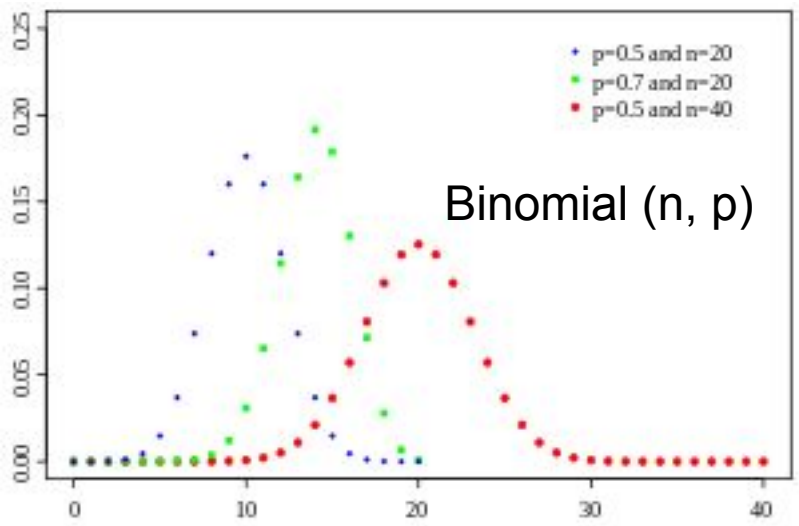

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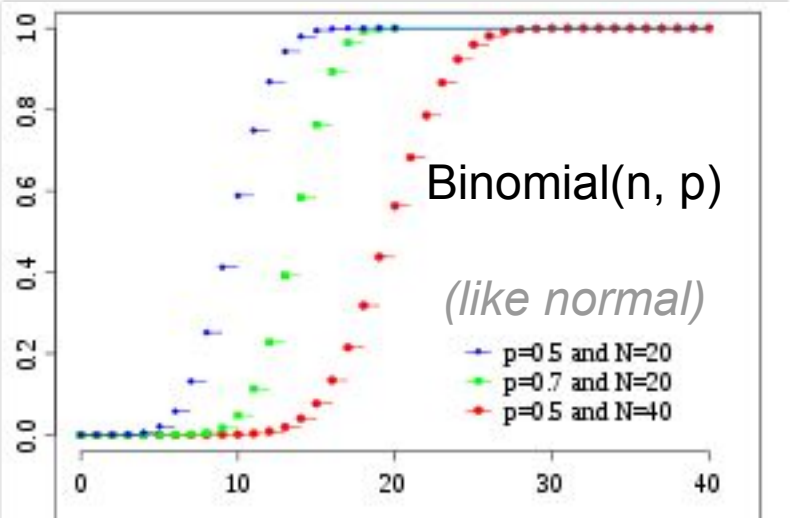

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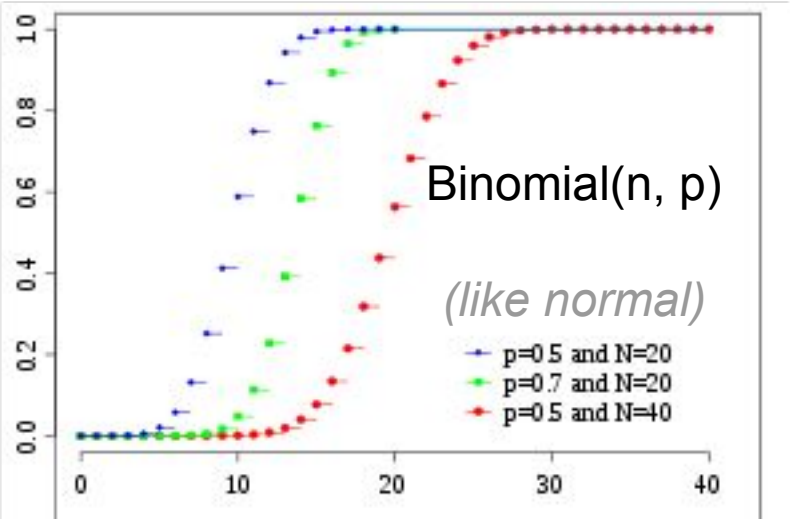
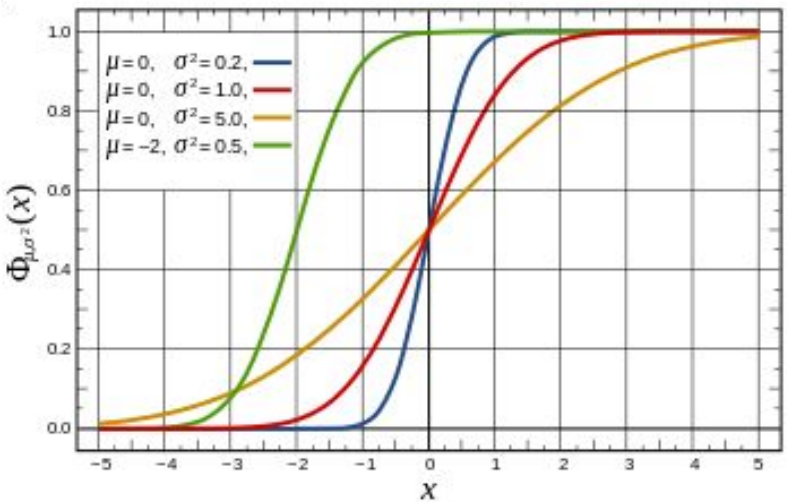


Normal



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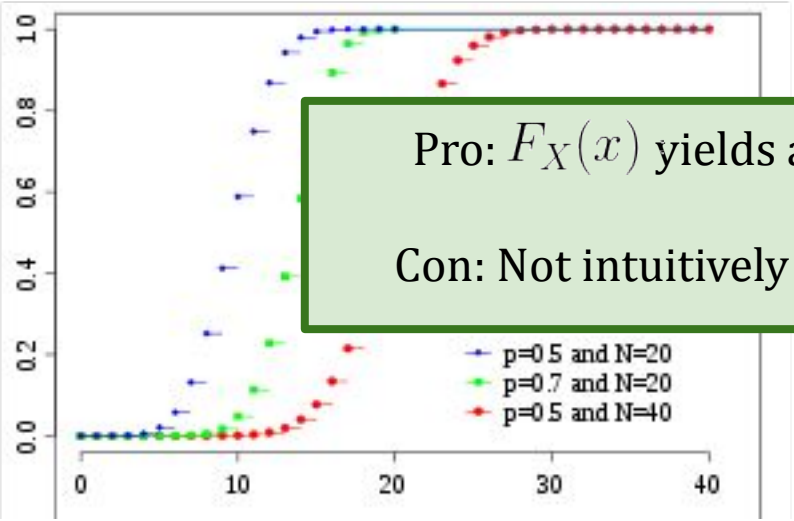
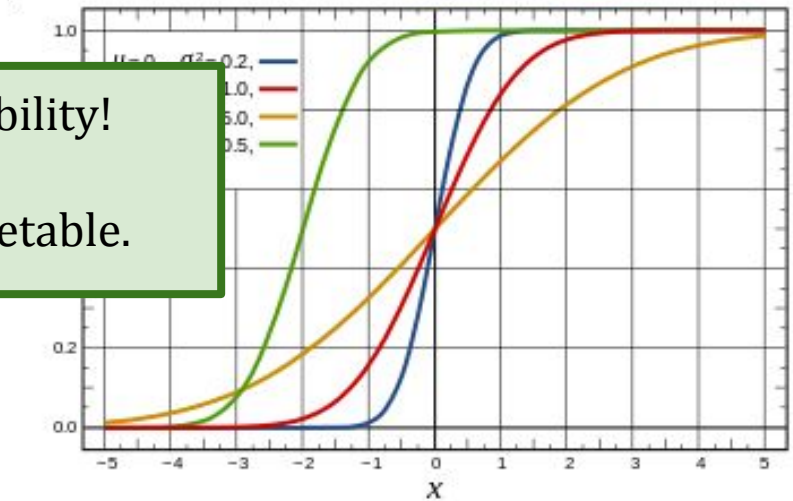


Pro: $F_X(x)$ yields a probability!

Con: Not intuitively interpretable.

Normal



Discrete RVs

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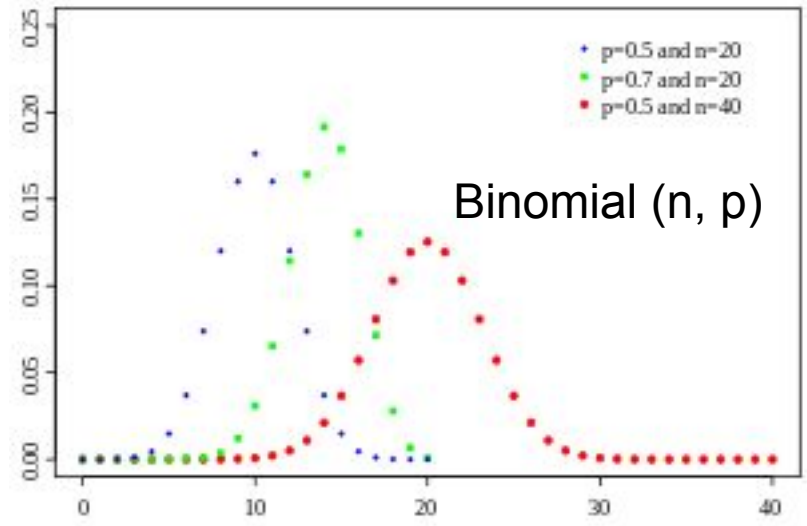

X is a *discrete random variable* if it takes only a countable number of values.

$$\sum_i f_X(x) = 1$$

$$F_X(x) = \mathrm{P}(X \leq x) = \sum_{x_i \leq x} f_X(x)$$

The Hypothesis Test "Algorithm"

```
Input: H0, observations, α
```

Need to estimate

What is the distribution of values we would expect if the null was true?
-- the "null distribution"

```
if p(x>=obs | H0) < α:
    decision = "Reject H0!"
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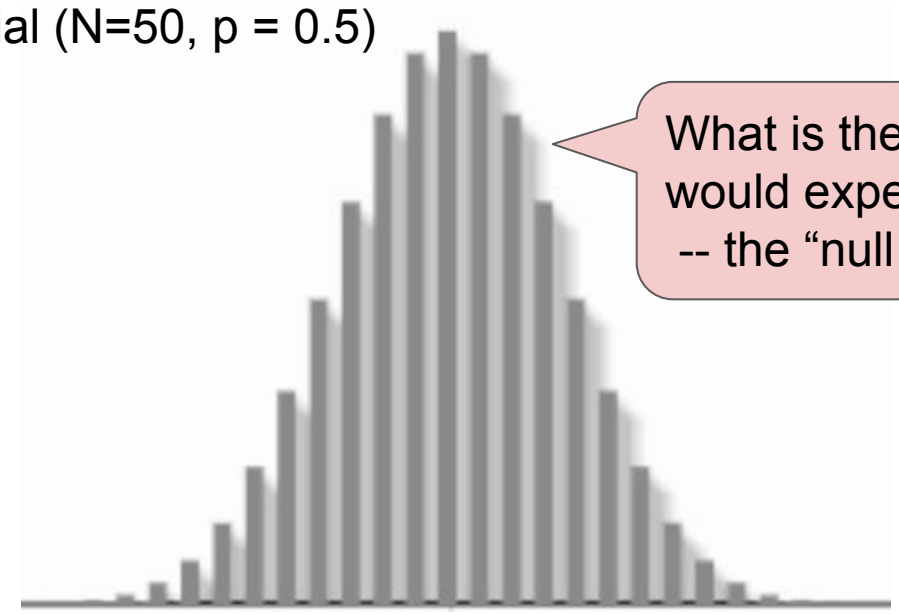
H_0: The blue case is not selling more than average.

The Hypothesis Test "Algorithm"

Binomial (N=50, p = 0.5)
PMF



What is the distribution of values we would expect if the null was true?
-- the "null distribution"

25

H_0: The blue case is not selling more than average.

50 sales; 2 colors (blue and red); Thus, average would be 25 blue sales

The Hypothesis Test "Algorithm"

```
Input: H0, obs, α

null_dist = distribution of expected values under H0



if p(x>=obs | H0) < α:
    decision = "Reject H0!"
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H_0: The blue case is not selling more than average. **Observed 33 blue sales**

50 sales; 2 colors (blue and red); Thus, average would be 25 blue sales

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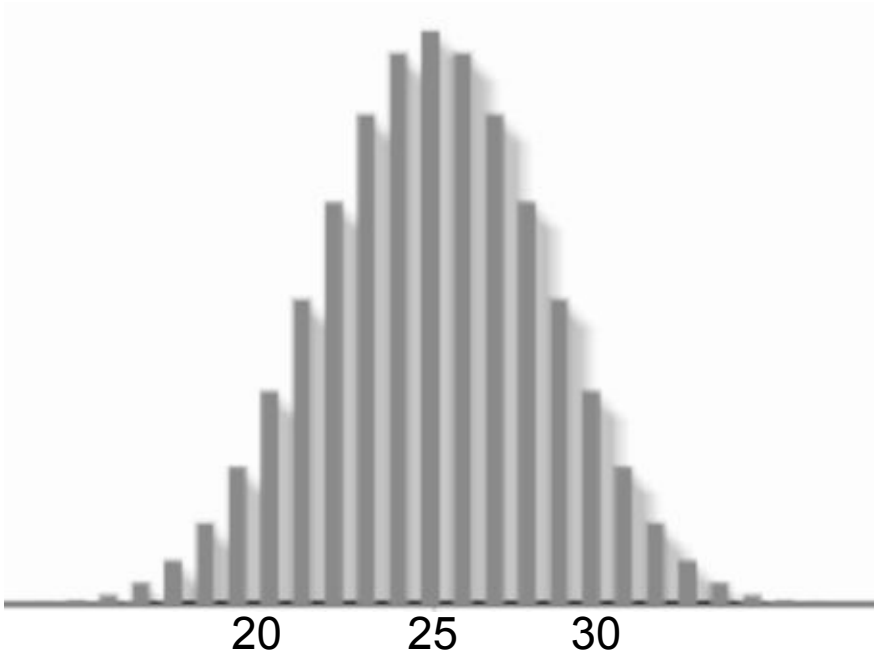

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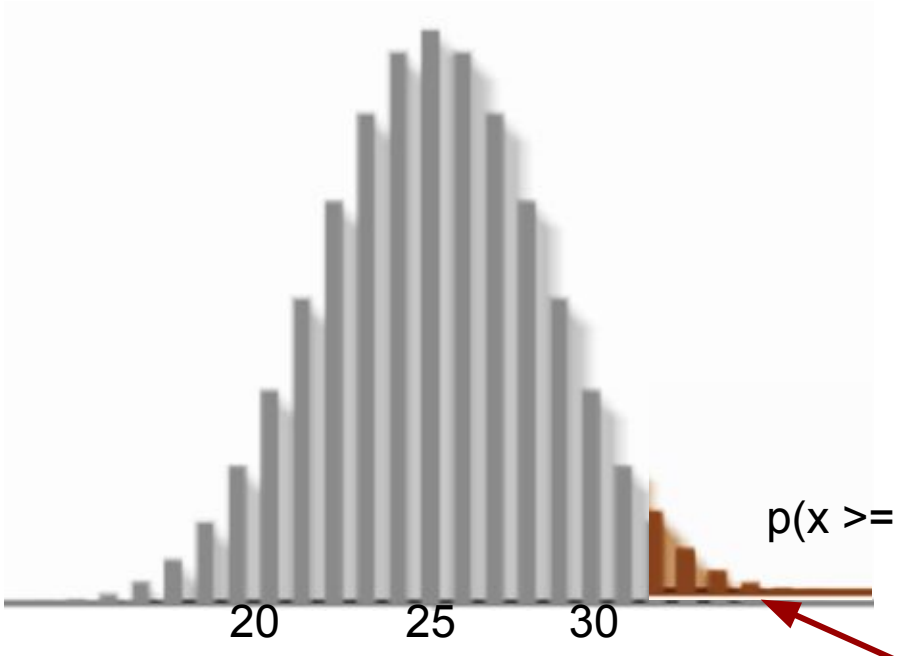

p(x >= 33; Binomial(50, 0.5)) = 0.016

H_0: The blue case is not selling more than average. **Observed 33 blue sales**

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Input: H0, obs, α

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p(x>=obs | H0) =

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```
Input: H0, obs, α

null_dist = distribution of expected values under H0

p(x>=obs | H0) = sum(pmf(null_dist, o) for o in range(obs,))

if p(x>=obs | H0) < α:
    decision = "Reject H0!"
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The Hypothesis Test "Algorithm"

```
Input: H0, obs, α

null_dist = distribution of expected values under H0

p(x>=obs | H0) = 1 - cdf(null_dist, obs-1)

if p(x>=obs | H0) < α:
    decision = "Reject H0!"
else:
    decision = "Accept H0."
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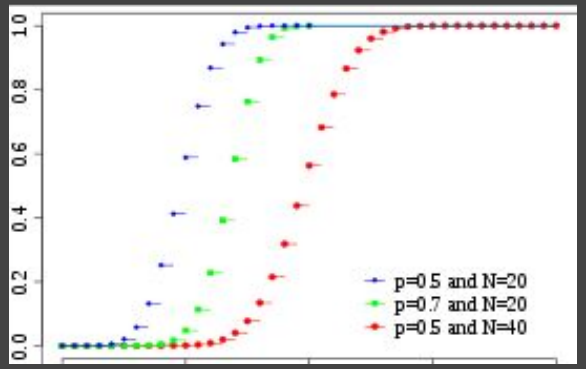

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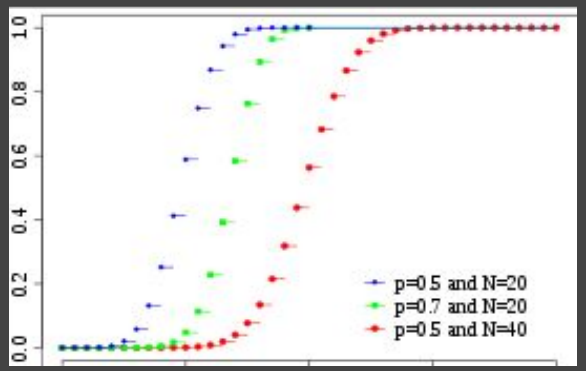

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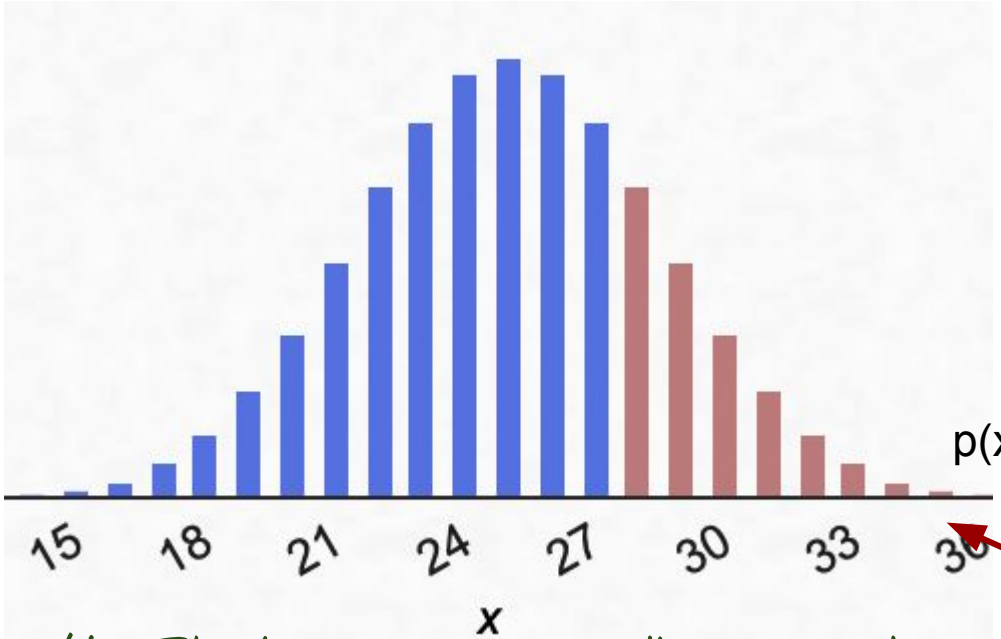

p(x >= 28; Binomial(50, 0.5)) = 0.239

H_0: The blue case is not selling more than average. **Observed 28 blue sales**

50 sales; 2 colors (blue and red); Thus, average would be 25 blue sales

The Hypothesis Test "Algorithm"

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Input: H0, obs, α

null_dist = distribution of expected values under H0

p(x>=obs | H0) = 1 - cdf(null_dist, obs-1) = 0.239

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Output: decision
```

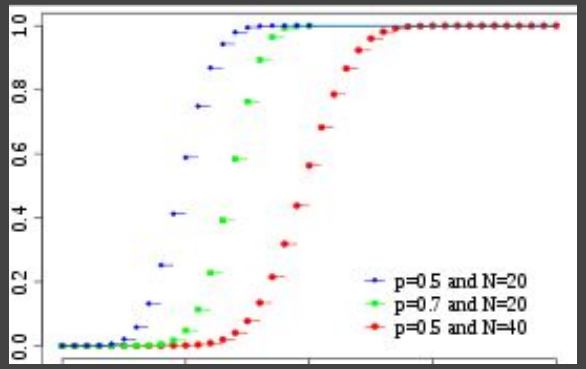

H_0: The blue case is not selling more than average. **Observed 28 blue sales**

50 sales; 2 colors (blue and red); Thus, average would be 25 blue sales

The Hypothesis Test "Algorithm"

```
Input: H0, obs, α

null_dist = distribution of expected values under H0

p(x<=obs | H0) = cdf(null_dist, obs)

if p(x<=obs | H0) < α:
    decision = "Reject H0!"
else:
    decision = "Accept H0."
Output: decision
```

H_0: The blue case is not selling **less than** average. Observed 32 blue sales

50 sales; 2 colors (blue and red); Thus, average would be 25 blue sales

The Hypothesis Test "Algorithm"

```
Input: H0, obs, α


 null_dist = distribution of expected obs under H0
 p(x<=obs | H0) = cdf(null_dist, obs)
 if p(x<=obs | H0) < α:
    decision = "Reject H0!"
 else:
    decision = "Accept H0."

Output: decision
```

H_0: The blue case is not selling **less than** average. Observed 36 blue sales

50 sales; 2 colors (blue and red); Thus, average would be 25 blue sales

The Hypothesis Test "Algorithm"

```
Input: H0, obs, α

 obs_ts = test_stat(obs)
 null_dist = distribution of expected obs under H0
 p(x<=obs | H0) = cdf(null_dist, obs)
 if p(x<=obs | H0) < α:
    decision = "Reject H0!"
 else:
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Output: decision
```

H_0: The blue case is not selling **less than** average. Observed 36 blue sales

50 sales; 2 colors (blue and red); Thus, average would be 25 blue sales

The Hypothesis Test "Algorithm"

```
Input: H0, obs, α

 obs_ts = test_stat(obs)
 null_dist = distribution of expected test_stat under H0
 p(x<=obs_ts | H0) = cdf(null_dist, obs_ts)
 if p(x<=obs_ts | H0) < α:
    decision = "Reject H0!"
 else:
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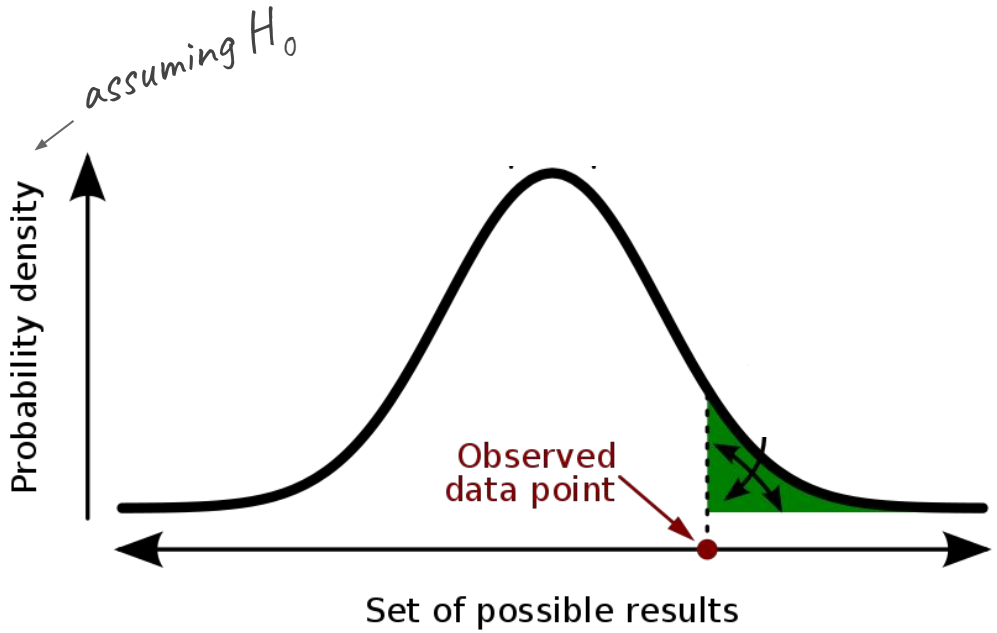
H_0: The blue case is not selling **less than** average. Observed 36 blue sales

50 sales; 2 colors (blue and red); Thus, average would be 25 blue sales

Hypothesis Testing

$P(D|H_0)$: *Given null, what is the probability of the observed data or worse?*

-> If <u>low enough</u>, then we "reject the null (H_0) in favor of H_1."



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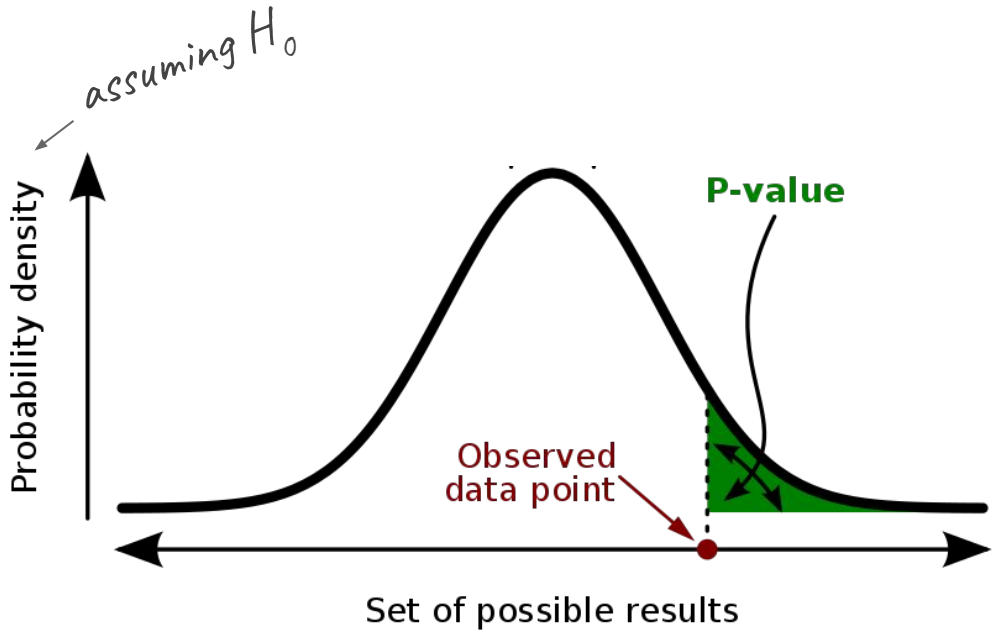

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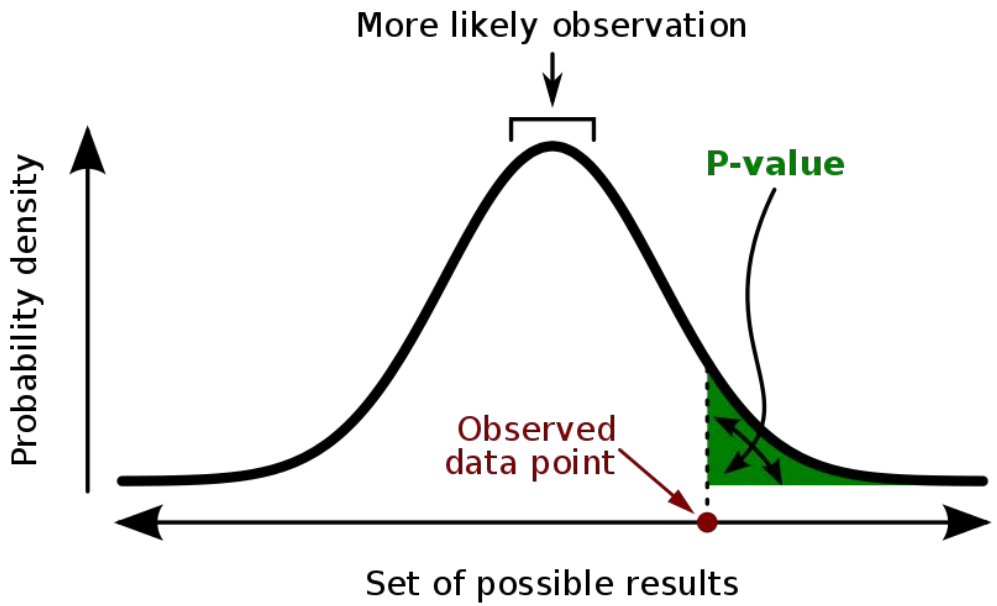

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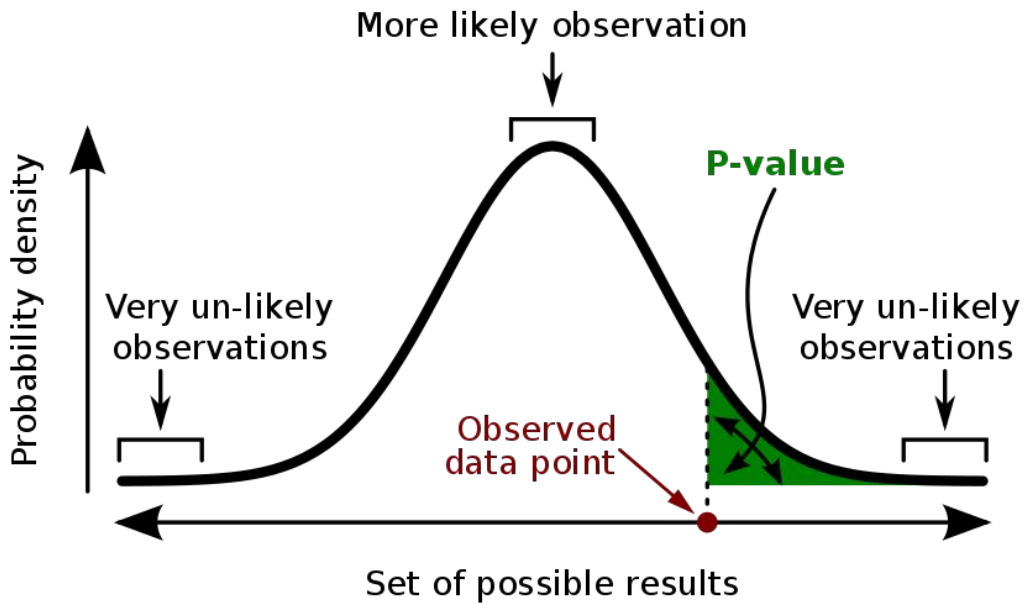

A **p-value** (shaded green area) is the probability of an observed (or more extreme) result assuming that the null hypothesis is true.

(thanks, Wikipedia)

The Hypothesis Test "Algorithm"

```
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 null_dist = distribution of expected test_stat under H0
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Hypothesis Testing

Why?

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A general framework for answering (yes/no) questions!

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- *Is my deep predictive model better than the state of the art?*

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Why?

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- *Are height and baldness related?*
- *Is my deep predictive model better than the state of the art?*
- *Is the heat index of a community related to poverty?*
- *Is the heat index of a community related to poverty* ***controlling for education rates****?*
- *Does my website receive a higher average number of monthly visitors?*

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Failing to "reject the null" does not mean the null is true.

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Hypothesis Testin

Failing to "reject the null" does not mean the null is true. However, if the sample is large enough, it may be enough to say that the effect size (correlation, difference value, etc...) is not very meaningful.

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A general framework for answering (yes/**maybe**) questions!

- *Are height and baldness related?*
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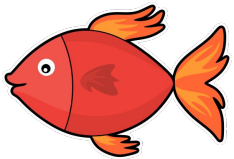
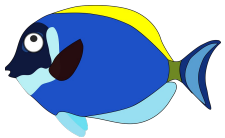
Bonferroni's Cats

General Question: Which fish do cats like?





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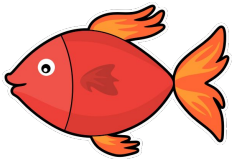

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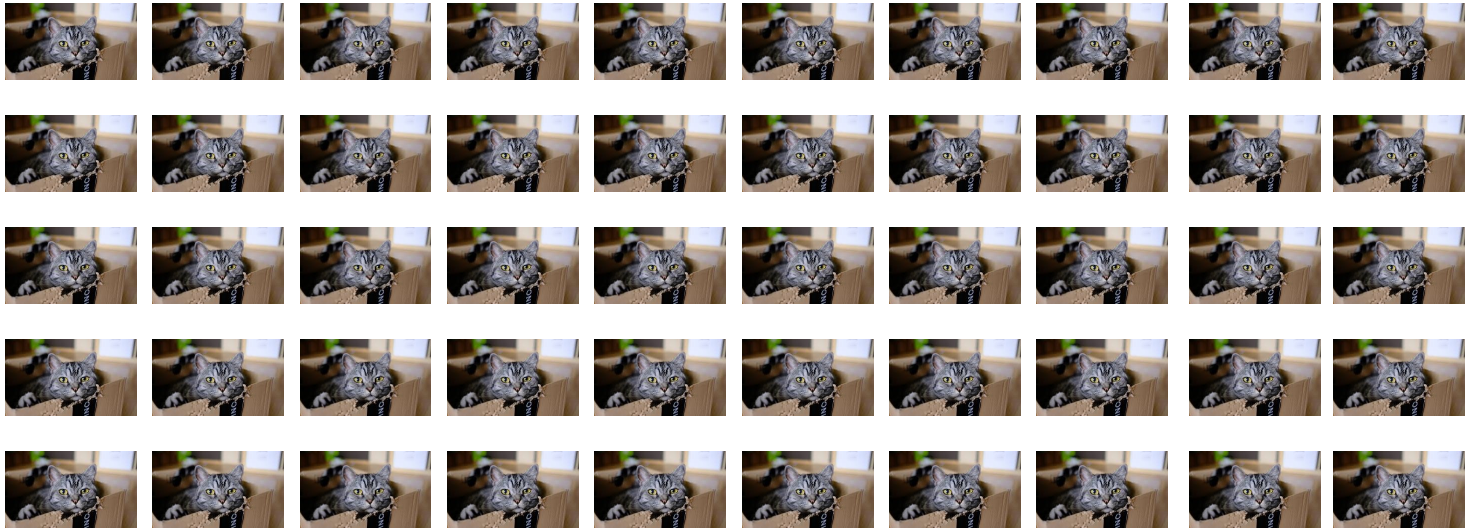

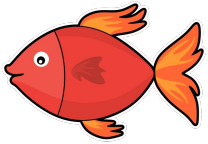

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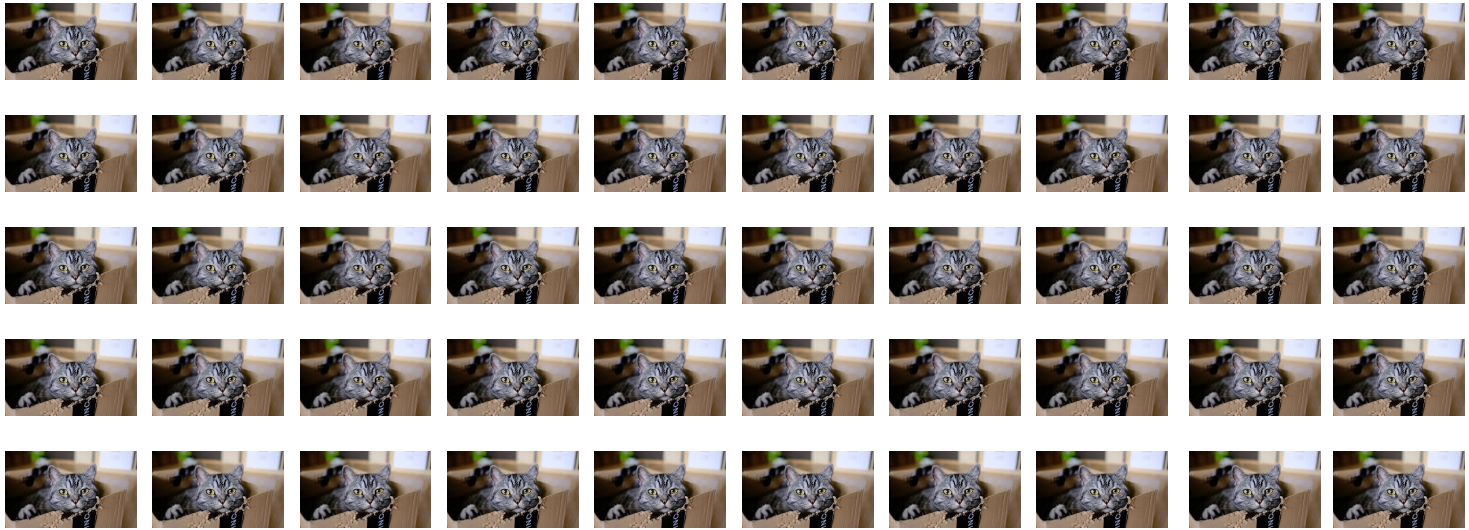

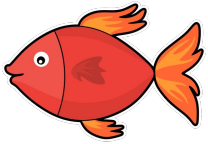

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H_1: Most cats like redfish. H_0: Most cats don't like redfish.

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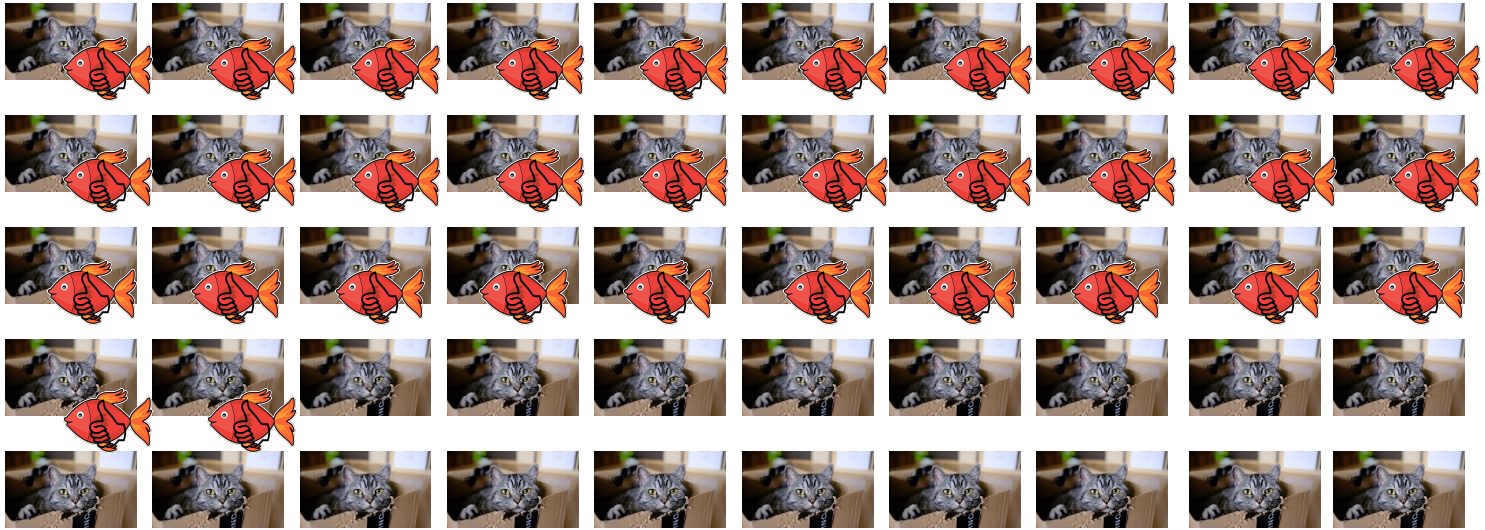
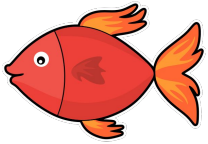
N = 50 cats; *32 like redfish; p = 0.016*





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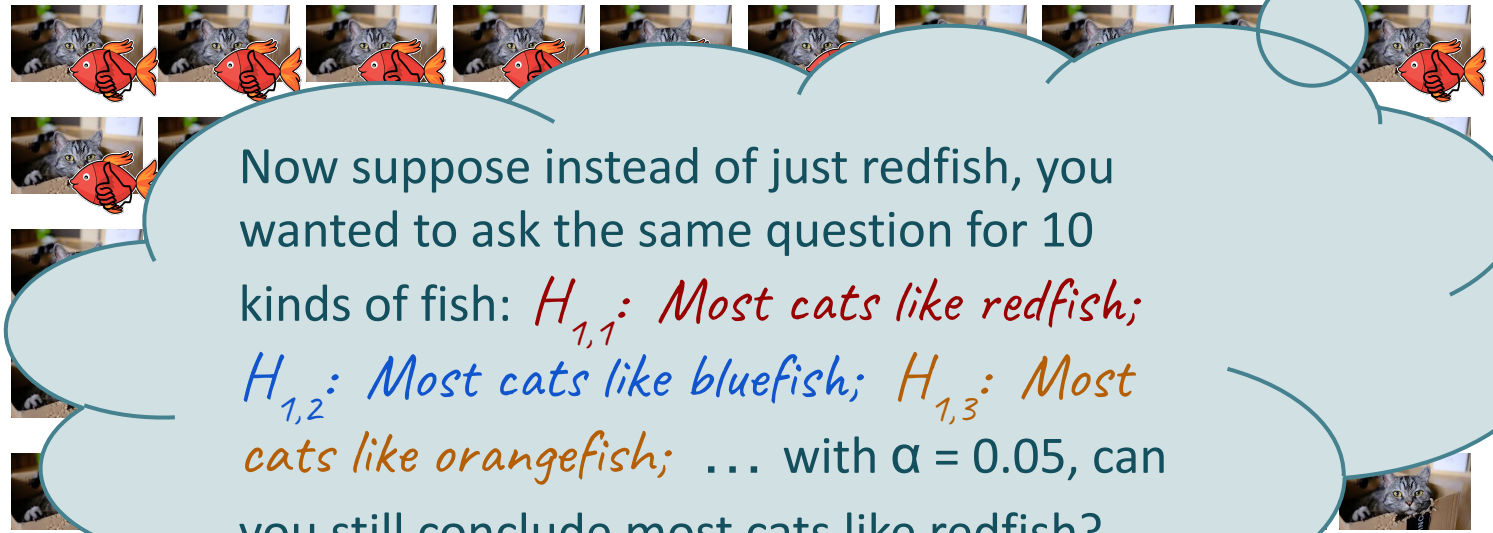

Now suppose instead of just redfish, you wanted to ask the same question for 10 kinds of fish: $H_{1,1}$: *Most cats like redfish;* $H_{1,2}$: *Most cats like bluefish;* $H_{1,3}$: *Most cats like orangefish;* … with α = 0.05, can you still conclude most cats like redfish?

H_1: *Most cat* … *don't like redfish.*

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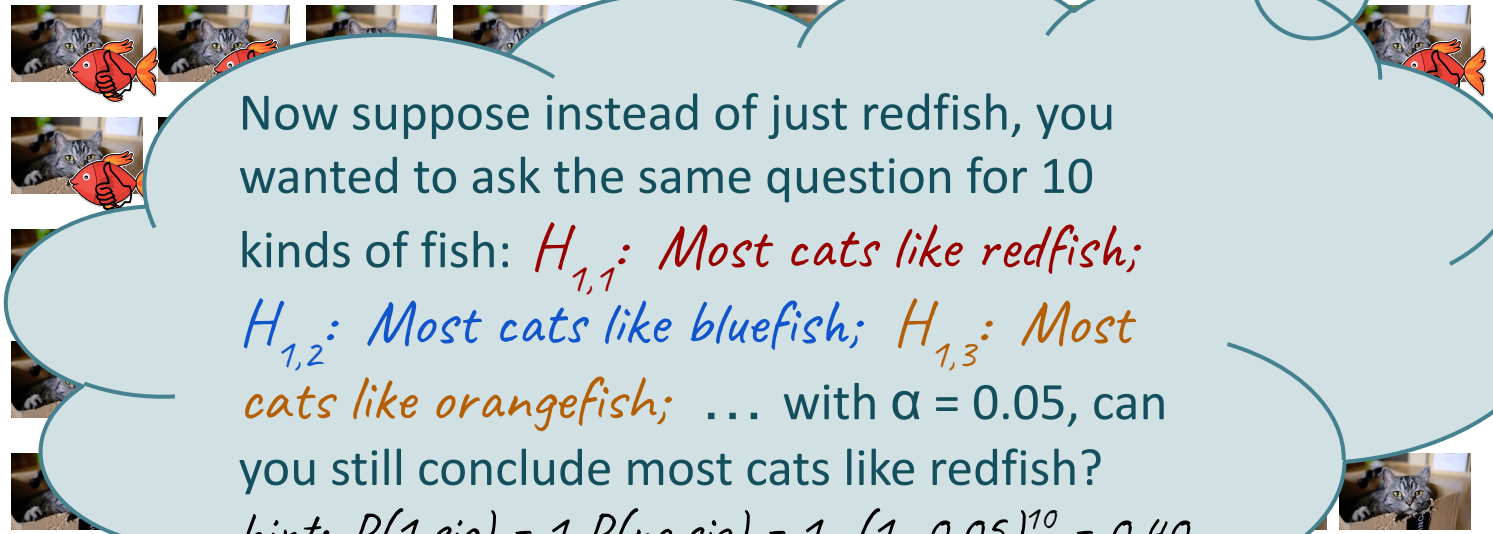

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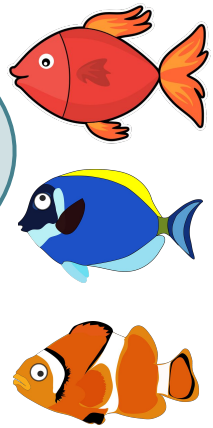
hint: $P(1\ sig) = 1 - P(no\ sig) = 1 - (1 - 0.05)^{10} = 0.40$



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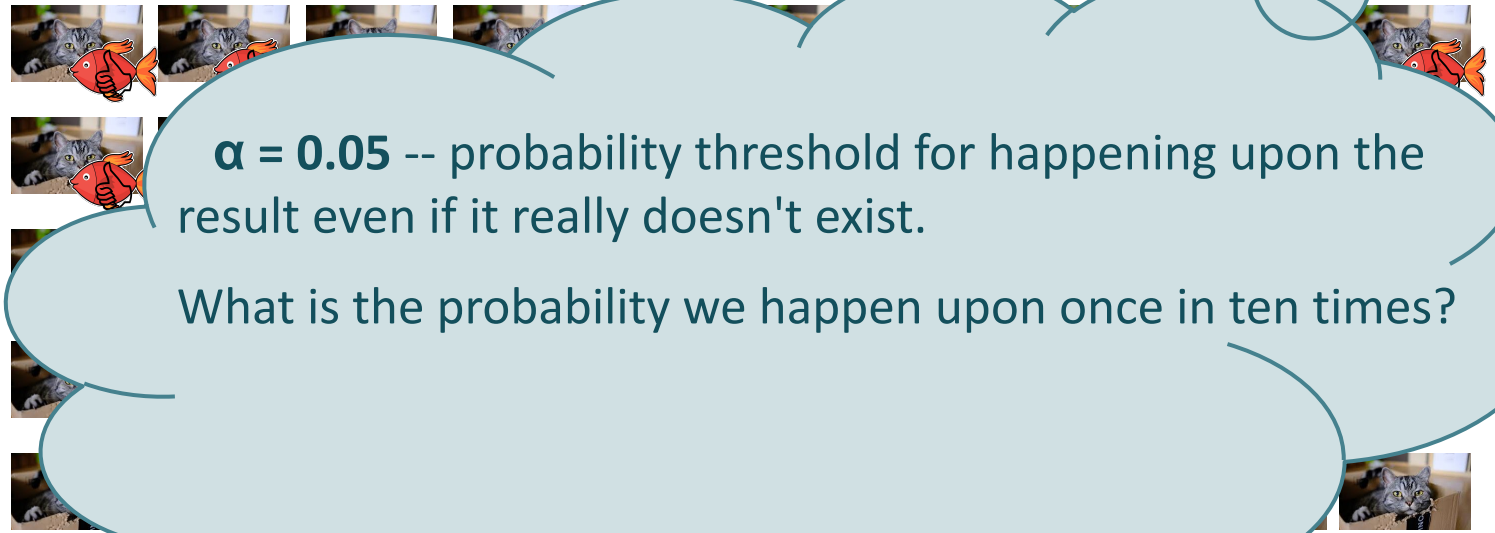
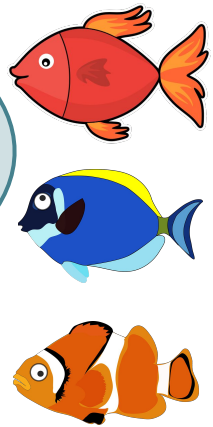


α = 0.05 -- probability threshold for happening upon the result even if it really doesn't exist.

What is the probability we happen upon once in ten times?



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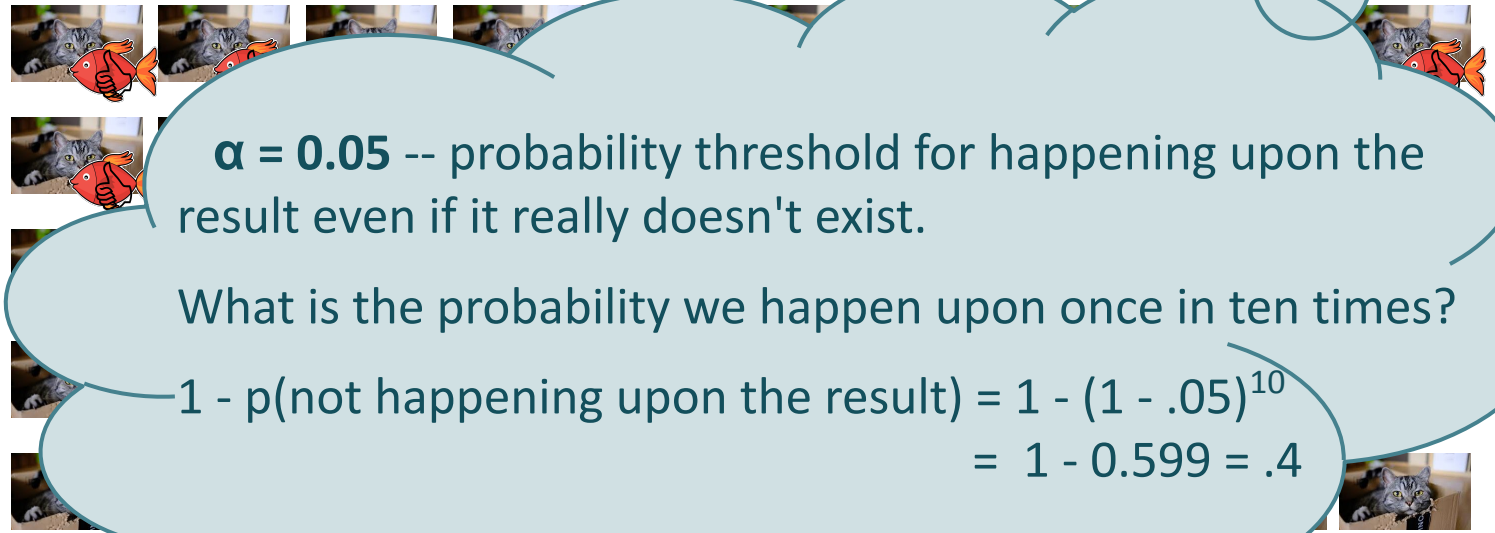

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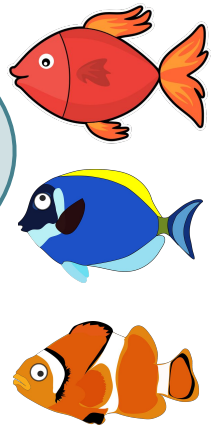
1 - p(not happening upon the result) = $1 - (1 - .05)^{10}$

= 1 - 0.599 = .4



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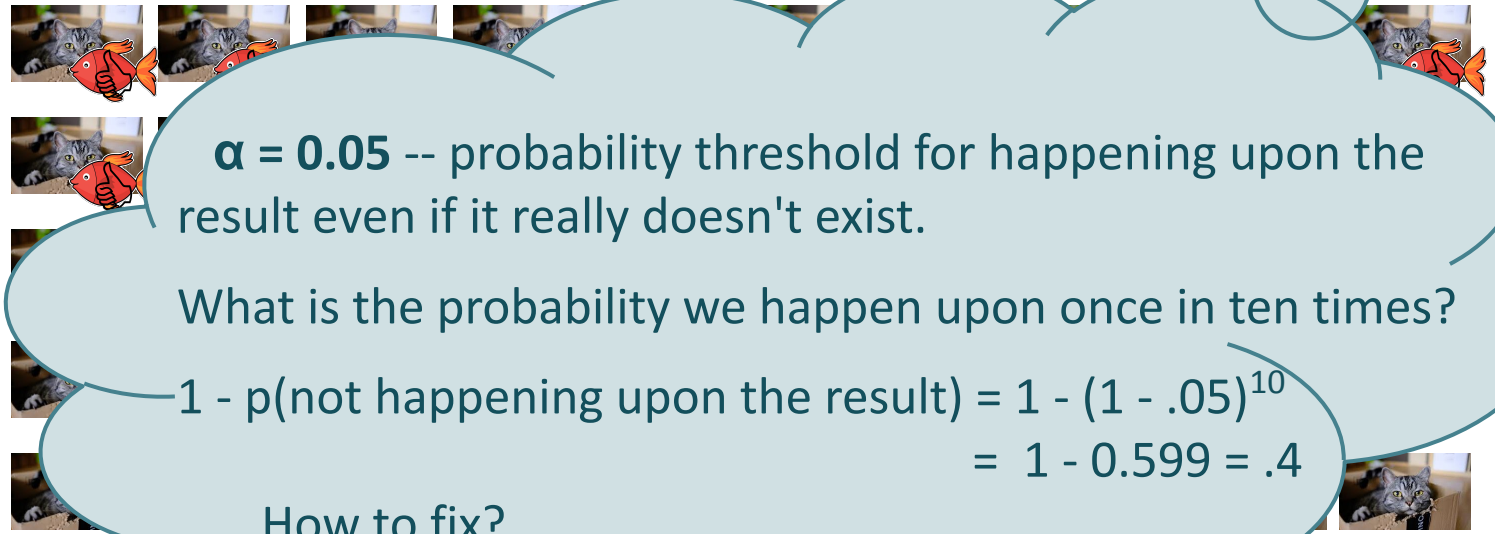

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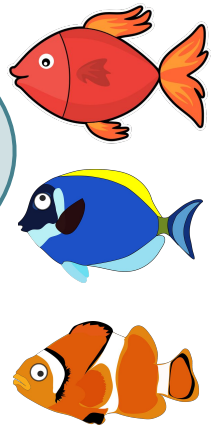

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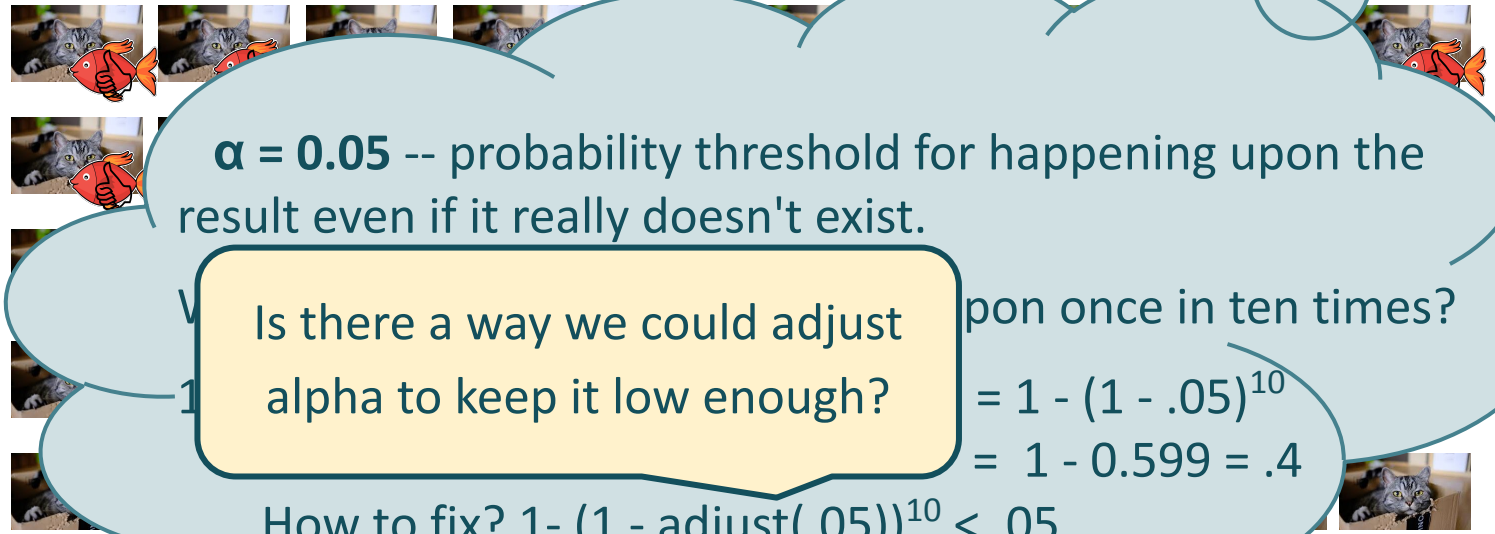

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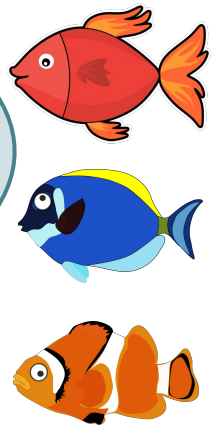

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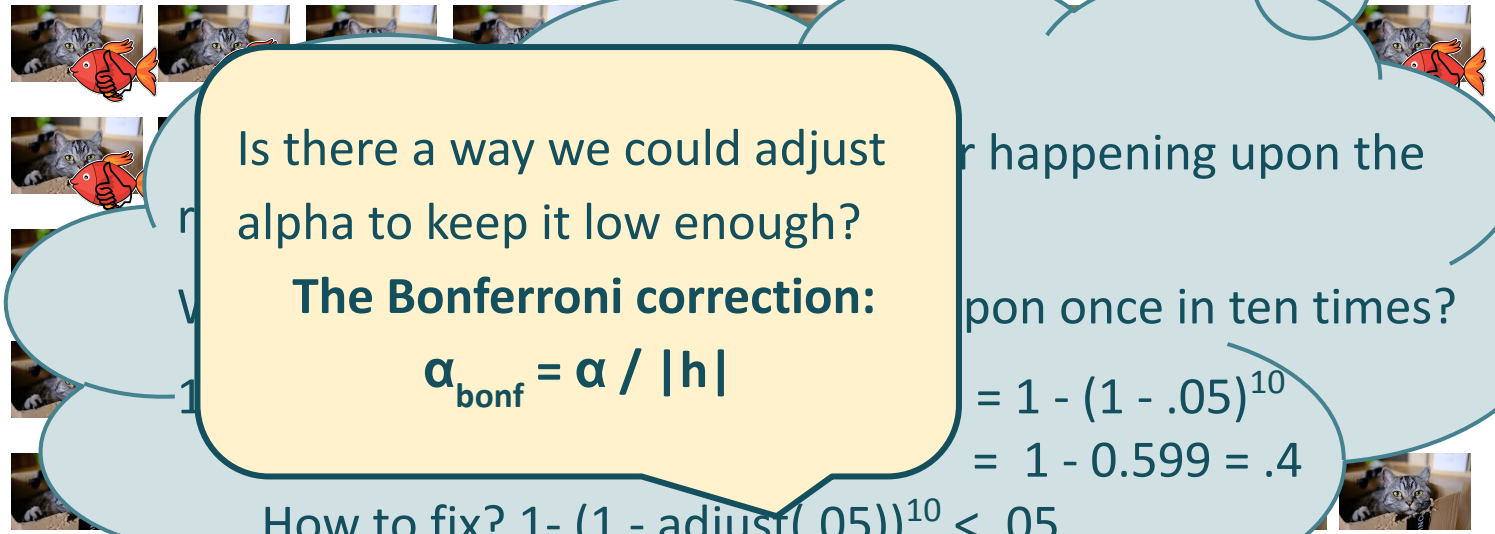

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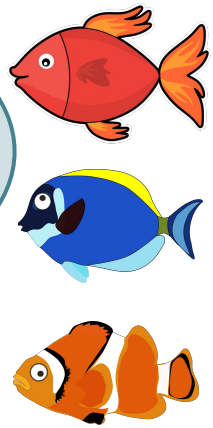
The Bonferroni correction:

$$\alpha_{bonf} = \alpha / |h|$$



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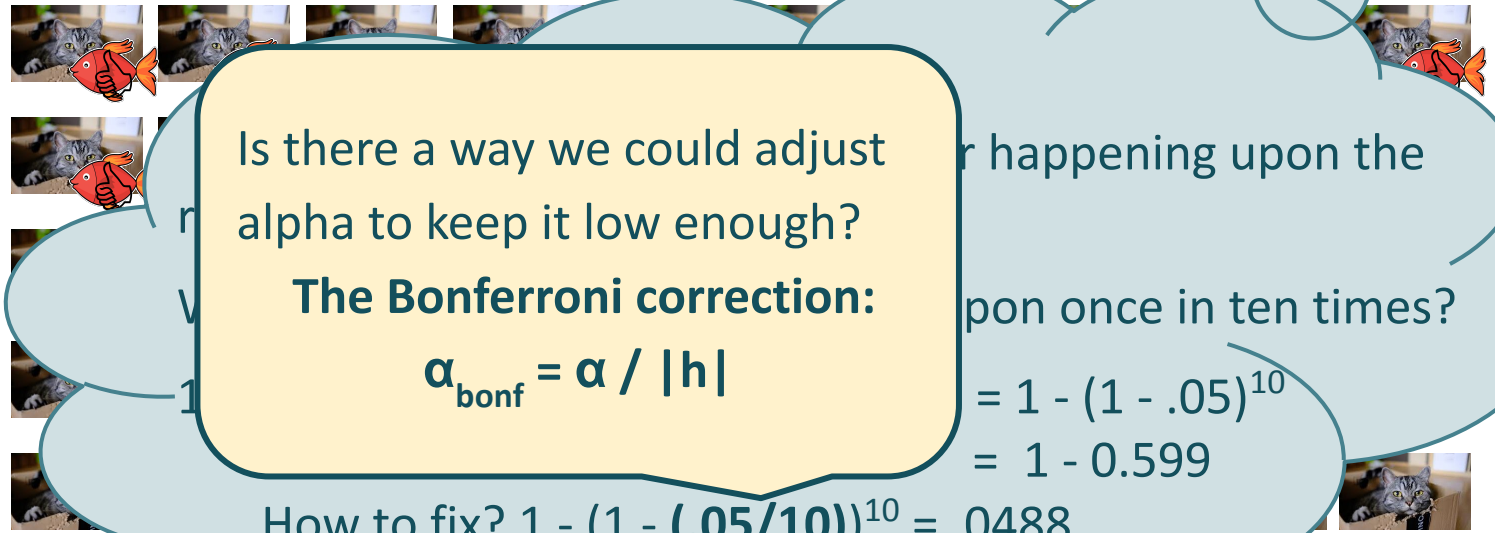

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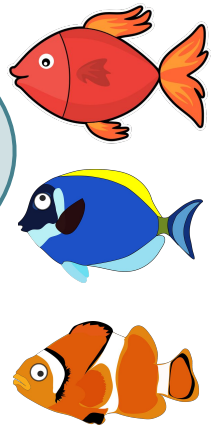

H_1: Most cats … don't like redfish.

Multi-test Correction

Type I, Type II Errors

		True state of nature	
		H_0	H_A
Our decision	Reject H_0	Type I error	correct decision
	'Accept' H_0	correct decision	Type II error

(Orloff & Bloom, 2014)

Multi-test Correction

significance level ("p-value") = P(type I error) = **P(Reject H_0 | H_0)**
(probability we are incorrect)

	H_0	H_A
Reject H_0	**P(Reject H_0 | H_0)**	

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(Orloff & Bloom, 2014)

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significance level ("p-value") = P(type I error) = **P(Reject H_0 | H_0)**
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power = 1 - P(type II error) = **P(Reject H_0 | H_1)**
(probability we are correct)

	H_0	H_A
Reject H_0	**P(Reject H_0 | H_0)**	**P(Reject H_0 | H_A)**

		True state of nature	
		H_0	H_A
Our decision	Reject H_0	Type I error	correct decision
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(Orloff & Bloom, 2014)

Multi-test Correction

FWER: Family-wise error rate (Bonferroni Corrects)

The probability of making >=1 type 1 error.

$$FWER = Pr(\text{type1s}>0) = 1 - Pr(\text{type1s}=0) = 1 - (1 - a)^m$$

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$$1 - (1 - \mathbf{(.05/10)})^{10} = .0488$$

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FDR: False discovery rate (Benjamini-Hochberg corrects)

type1s / (type1s + correctRejects)

		True state of nature	
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Our decision	Reject H_0	Type I error	correct decision
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FDR: False discovery rate (Benjamini-Hochberg corrects)

type1s / (type1s + correctRejects)

Proportion of false positives among *all* significant results.

(Orloff & Bloom, 2014)

The Hypothesis Test "Algorithm"

```
Input: H0, obs, α

 obs_ts = test_stat(obs)
 null_dist = distribution of expected test_stat under H0
 p(x<=obs_ts | H0) = cdf(null_dist, obs_ts)
 if p(x<=obs_ts | H0) < α:
    decision = "Reject H0!"
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Output: decision
```

The Multi-test "Algorithm"

```
Input: H0s, obs, α

decisions = []
α_adj = adjust(α)

for H0 in H0s
   obs_ts = test_stat(obs)
   null_dist = distribution of expected test_stat under H0
   p(x<=obs_ts | H0) = cdf(null_dist, obs_ts)
   if p(x<=obs_ts | H0) < α_adj:
      decisions.append("Reject H0!")
   else:
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Output: decisions
```

The Multi-test "Algorithm"

```
Input: H0s, obs, α

decisions = []
α_adj = adjust(α) #e.g. adjust(α) = α/len(H0s)

for H0 in H0s
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   p(x<=obs_ts | H0) = cdf(null_dist, obs_ts)
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```

Multi-test "Algorithm" **Alternative**

```
Input: H0s, obs, α

decisions = []
for H0 in H0s
   obs_ts = test_stat(obs)
   null_dist = distribution of expected test_stat under H0
   p(x<=obs_ts|H0) = cdf(null_dist, obs_ts)
   p_adj = inverse_adjust(p(x<=obs_ts|H0))#e.g. p*len(H0s)
   if p_adj < α:
      decisions.append("Reject H0!")
   else:
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Output: decisions
```

Multiple Linear Regression

Simple Linear Regression $$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

where $\mathbf{E}(\epsilon_i | X_i) = 0$ *and* $\mathbf{V}(\epsilon_i | X_i) = \sigma^2$

expected variance

Estimated intercept and slope

$$\hat{r}(x) = \hat{\beta}_0 + \hat{\beta}_1 x \qquad \hat{Y}_i = \hat{r}(X_i)$$

Residual: $\hat{\epsilon}_i = Y_i - \hat{Y}_i$

Multiple Linear Regression

Suppose we have multiple X that we'd like to fit to Y at once:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_m X_{m1} + \epsilon_i$$

If we include and $X_{oi} = 1$ for all i (i.e. adding the intercept to X), then we can say:

$$Y_i = \sum_{j=0}^{m} \beta_j X_{ij} + \epsilon_i$$

Multiple Linear Regression

Suppose we have multiple X that we'd like to fit to Y at once:

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If we include and $X_{oi} = 1$ for all i, then we can say:

$$Y_i = \sum_{j=0}^{m} \beta_j X_{ij} + \epsilon_i$$

Or in vector notation across all i:

$$Y = X\beta + \epsilon$$

where β and ϵ are vectors and X is a matrix.

Multiple Linear Regression

Suppose we have multiple X that we'd like to fit to Y at once:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_m X_{m1} + \epsilon_i$$

If we include and $X_{oi} = 1$ for all i, then we can say:

$$Y_i = \sum_{j=0}^{m} \beta_j X_{ij} + \epsilon_i$$

Or in vector notation across all i:

$$Y = X\beta + \epsilon$$

where β and ϵ are vectors and X is a matrix.

Estimating β :

$$\hat{\beta} = (X^T X)^{-1} X^T Y$$

Multiple Linear Regression

Suppose we have multiple independent variables that we'd like to fit to our dependent variable: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_m X_{m1} + \epsilon_i$

If we include and $X_{oi} = 1$ for all i. Then we can say:

$$Y_i = \sum_{j=0}^{m} \beta_j X_{ij} + \epsilon_i$$

To test for significance of individual coefficient, j:

$$t = \frac{\hat{\beta}_j}{SE(\hat{\beta}_j)} = \frac{\hat{\beta}_j}{\sqrt{\frac{s^2}{\sum_{i=1}^{n}(X_{ij} - \bar{X}_j)^2}}}$$

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Significance Testing

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_m X_{m1} + \epsilon_i$$

$$s^2 = \frac{RSS}{df}$$

To test for significance of individual coefficient, *j*:

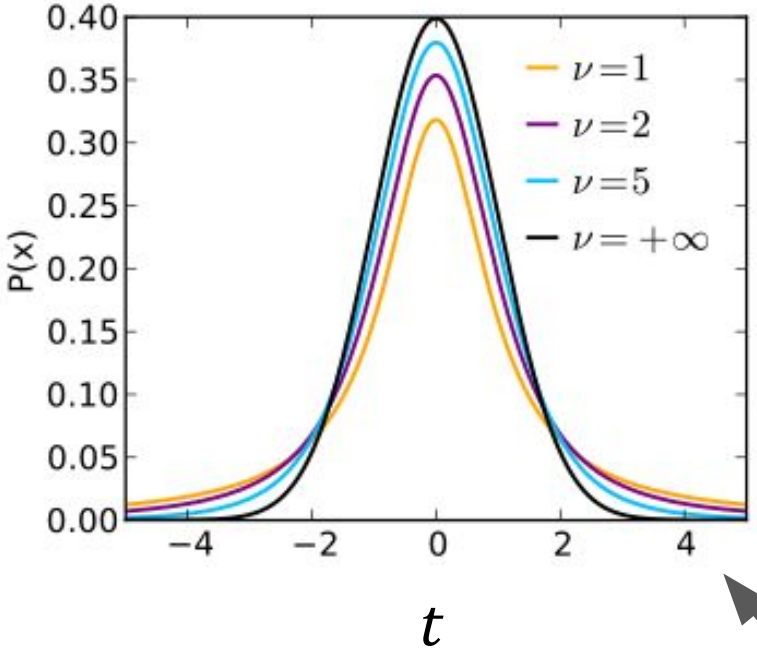
$$t = \frac{\hat{\beta}_j}{SE(\hat{\beta}_j)} = \frac{\hat{\beta}_j}{\sqrt{\frac{s^2}{\sum_{i=1}^{n}(X_{ij} - \bar{X}_j)^2}}}$$

T-Test for significance of hypothesis:

1) Calculate t
2) Calculate degrees of freedom:

$$df = N - (m+1)$$

3) Check probability in a *t* distribution:



$$\beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_m X_{m1} + \epsilon_i$$

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<u>T-Test for significance of hypothesis:</u>

1) Calculate t
2) Calculate degrees of freedom:

$$df = N - (m+1)$$

3) Check probability in a t distribution: ($df = v$)

Summary: Hypothesis Testing

Hypothesis Testing:

A framework for deciding which differences/relationships matter.

- Random Variables
- Distributions
- Hypothesis Testing Framework

Comparing Variables:

Metrics to quantify the difference or relationship between variables.

- Simple Linear Regression, Correlation, Multiple Linear Regression,
- Comparing Variables and Hypothesis Testing
- Regularized Linear Regression (for supervised ML)
- Multiple Hypothesis Testing

Statistical Standards

1. Correct for multiple tests (Bonferonni's Principle)
2. Average multiple models (ensemble techniques)
3. Smooth data
4. "Plot" data (or figure out a way to look at a lot of it "raw")
5. Interact with data

(http://simplystatistics.org/2014/05/22/10-things-statistics-taught-us-about-big-data-analysis/)

Statistical Standards

1. Correct for multiple tests (Bonferonni's Principle)
2. Average multiple models (ensemble techniques)
3. Smooth data
4. "Plot" data (or figure out a way to look at a lot of it "raw")
5. Interact with data
6. Know your "real" sample size
7. Correlation is not causation
8. Define metrics for success (set a baseline)
9. Share code and data
10. The problem should drive solution

(http://simplystatistics.org/2014/05/22/10-things-statistics-taught-us-about-big-data-analysis/)

Large-Scale Hypothesis Testing

- Findings and Uncertainty

- Hypothesis Testing

- Bonferroni's Cats
- Multi-test Corrections
  - Family-wise Error Rate
  - False-Discovery Rate
- Correlation Metrics
  - Effect Size (coefficient)
  - Significance (whether p-value is below significance level)